Measuring macroeconomic volatility

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Summary

The literature on macroeconomic volatility covers an extremely wide field, as evidenced by the very broad spectrum of indicators used to grasp it as a phenomenon. In general there seems to be little discussion about the choice of indicator for macroeconomic volatility, on the grounds that the different methods, based on stationary series, give rise to scores which are closely correlated. Although these methods may converge when analysing only the average magnitude of distribution around a reference value, however, they diverge significantly when one examines either asymmetry or kurtosis (instances of extreme deviation). This article reviews the existing literature and sets out the principal methods used for calculating volatility. These methods are compared and their properties analysed based on export revenue data for 134 countries from 1970 to 2005. In particular, a distinction is drawn between measurements of the magnitude of volatility and measurements of asymmetry and the incidence of extreme deviations.

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1. Introduction

The numerous global economic crises of the 20th century have made macroeconomic volatility a key issue in analysing the determinants of economic growth. The multiplicity of ways in which it affects the long-term growth potential of economies, its diverse causes and the array of methods by which it is measured, make economic volatility a complex and multidimensional phenomenon. We therefore consider the term “volatility” as a generic term, combining all the techniques available for measuring economic fluctuations.

1.1. Macroeconomic volatility and development.

The literature provides an extensive analysis of the costs and consequences of macroeconomic volatility. Although the positive relationship between risk and capital yield may, under certain conditions, explain a positive relationship between economic volatility and growth, most research agrees that this phenomenon has a negative impact on long-term growth and well-being. Indeed, over the long term, volatility contributes to a reduction in levels of consumption, investment and factor productivity, to an increase in the volatility and unpredictability of economic policy, and to deterioration in the institutional environment. The effects on performance are even more marked in developing countries, which are often subjected to more significant external shocks but which do not enjoy the internal conditions that would allow them to absorb them more easily. Hnatkovska and Loayza (2005), Aizenman and Pinto (2005) and Loayza et al (2007) offer an exhaustive overview of the consequences of macroeconomic volatility and the factors that cause it.

The costs of volatility

Macroeconomic volatility is a major obstacle to growth. According to estimates produced by Hnatkovska and Loayza (2005), based on a sample of 79 countries, increasing the average value of volatility by the value of its standard deviation results in an average loss of 1.3 points for growth in GDP over the period 1960-2000, and 2.2 points for the decade 1990-2000. Volatility can, indeed, act as an obstacle to the key factors in economic and social development.

An early series of research articles examined the impact of macroeconomic volatility on growth from the point of view of investment or production factor productivity. Dawe (1996) analyses the effect of volatility in exports on investment and growth. He finds both a positive effect of volatility on investment through an increase in precautionary savings, and a negative effect on growth through an allocation of capital to sectors with lower yields. Dehn (2000), on the other hand, identifies a significant and negative impact of shocks in the price of raw materials on investment in developing countries. Guillaumont, Guillaumont-Jeanneney, and Brun (1999) highlight a negative effect of volatility in the rate of investment on growth, based on a decline in average productivity. In a similar area, Koren and Tenreyro (2006) empirically test a theoretical model where development is accompanied by increased diversification of inputs into the production system, thus reducing the effect of volatility in world prices on production factor productivity and therefore on growth. Finally, Combes and Guillaumont (2002) show that volatility in relation to terms of trade has a
negative impact on the growth rate of capital productivity. Macroeconomic volatility therefore seems to be an obstacle to economic growth insofar as it discourages investment decisions, has a negative effect on factor productivity and diverts capital from the most productive sectors.

Other studies analyse the impact of macroeconomic volatility on growth and well-being through its effect on the quality of economic policy. Easterly et al (1993) show that positive shocks in relation to terms of trade influence the long-term growth path of economies, in part through an improvement in economic policy. Ramey and Ramey (1995) show that the unpredictability of economic policy caused by volatility in growth rates has a negative effect on the average growth rate of the economy. Guillaumont and Combes (2002) show that vulnerability to volatility in global prices has a negative effect on the quality of economic policy and growth. To take two other examples, both Fatas and Mihov (2006, 2007) and Afonso and Furceri (2010) have emphasised the negative impact of variability in budget policy on growth in both OECD countries and developing nations.

The negative effect of macroeconomic volatility on growth and well-being, based on volatility in public and private consumption, has also been examined in a number of studies. Aizenman and Pinto (2005) and Wolf (2005) point out that in the case of imperfect financial markets, the State and individual households are unable to protect themselves fully against risks which affect their revenue and adjust their consumption to the vagaries of economic activity. The result is that volatility driven by external factors, for example in relation to terms of trade, generates internal volatility in relation to consumption, particularly in developing countries (Aguiar and Gopinath, 2007; Loayza et al, 2007).

Economic policy and consumption therefore appear to be internal channels by which macroeconomic volatility is transmitted and may even be magnified, with a concomitant negative effect on growth and development.

**Internal economic conditions resulting in greater vulnerability to economic volatility.**

One positive effect of volatility on growth has already been mentioned (Imbs, 2007; Rancière et al, 2008; Hnatkovska and Loayza, 2005). This can be explained primarily by the positive correlation between risk and return on investment projects. Hnatkovska and Loayza (2005) suggest, however, that a positive effect of this kind is dependent on the existence of risk-sharing mechanisms and respect for rights of ownership, which are in turn supported by a well-developed financial system and high-quality institutions. A country’s vulnerability to macroeconomic volatility is therefore driven by a number of handicaps, which are either structural or depend on the level of economic development. These factors explain why, in general terms, developing countries are more vulnerable to macroeconomic volatility. Developing countries are more exposed to shocks, and do not always have the mechanisms or internal conditions in place to enable them to absorb them. The size of the population, the degree of diversification of the economy and the capacity for operating a countercyclical economic policy, the existence of well-developed financial institutions and institutional quality are therefore determining factors in the impact of volatility on growth.

The literature on economic vulnerability has made a significant contribution to our understanding
of the internal and external conditions of vulnerability to shocks (Guillaumont, 2007, 2009a, 2009b, 2010; Cariolle, 2011; Loayza and Raddatz, 2007; Combes and Guillaumont, 2002). The research carried out distinguishes structural factors in relation to vulnerability from more transitory factors linked to economic policy (or “resilience”). As far as structural factors are concerned, a distinction is drawn between the magnitude and frequency of shocks (commercial or natural) and exposure to such shocks. Factors that affect exposure to shocks (such as the size of the population, the degree of economic diversification, distance from global markets and geographical isolation) increase the propensity of economies to suffer shocks and the negative impact of such shocks on growth. A recent study by Malik and Temple (2009), on the structural determinants of volatility in relation to growth, suggests a negative relationship between access to global markets and macroeconomic volatility. According to the authors, countries which are isolated from global markets tend to lack diversity in terms of exports and experience greater volatility in relation to GDP. As to the resilience of particular countries, although this is to some extent dependent on structural factors, it is linked primarily to economic policy and institutions. As a result, development strategies with a focus on foreign trade, the procyclicality and countercyclicality of economic policy and the quality of governance and democratic institutions (Rodrik, 1998; 2000) can determine both the magnitude of volatility experienced by countries and its effect on their development.

The contribution made by foreign trade to macroeconomic volatility is one of the themes most commonly addressed in the literature. Whilst the role played by openness to trade has not been clearly established (Combes and Guillaumont, 2002), several pieces of research have examined the relationship between a country’s degree of specialisation, its level of development and macroeconomic volatility. Di Giovanni and Levchenko (2010) study the extent to which openness to trade can result in a specialisation of export sectors which are highly exposed to external shocks, and thus to increased economic volatility. They show that countries with a low or moderate comparative advantage in high-risk export sectors diversify their economy in order to attenuate the risk affecting their export revenues. Conversely, countries with a very high level of comparative advantage in these sectors tend to specialise in them, and are thus more exposed to volatility in relation to terms of trade, exports and GDP growth per head. Similarly, Koren and Tenreyro (2006, 2007) show that poor countries specialise in a limited number of sectors, with relatively simple production technologies and a limited range of inputs, and are therefore more vulnerable to shocks in global prices. Development is therefore supported by diversification into sectors based on more complex technologies, using a wider variety of inputs, and with less exposure to macroeconomic volatility. Van der Ploeg and Poelhekke (2009) find that growth volatility, driven by volatility in global raw materials prices, is the main determinant of the “natural resource curse”. Having controlled for growth volatility, they show that supplies of natural resources have a positive and significant effect on economic growth. Development is thus generally accompanied by economic diversification and by specialising in sectors which are less exposed to global volatility. Conversely, the negative effect of a low level of economic diversification on development is assumed to be dependent on a high level of exposure to volatility in global prices.
Other studies have examined the quality of institutions in attenuating or contributing to economic volatility. Acemoglu et al (2003) showed that poor institutions result in poor governance, which in turn contributes to macroeconomic volatility. Mobarak (2005) finds that democracy reduces volatility through increased scrutiny by citizens of the management of economic policy. These results reflect those found by Rodrik (2000), according to which democratic political structures encourage political consensus around political responses to external shocks. Numerous research articles have also examined the role of the development of financial markets in transmitting macroeconomic volatility (Beck et al, 2006; Aghion et al, 2005; Aghion et al, 2004). In general, financial development therefore tends to attenuate shocks, although it seems to be able to magnify the effect of shocks of monetary origin on volatility in GDP (Beck et al, 2006). The quality of political, economic and financial institutions therefore seems to be both a source and a vector of macroeconomic volatility.

**The origins of macroeconomic volatility**

Another set of literature examines the sources of macroeconomic volatility. This research generally draws a distinction between external forms of volatility (exports, global prices, terms of trade or international interest rates) and internal forms (such as economic policy, agricultural production and natural or climatic disasters). Similarly, it is possible to distinguish between exogenous sources of macroeconomic volatility (related to international trade, agricultural production and natural disasters) and endogenous sources (linked to volatility in economic policy or domestic socio-political conditions). Finally, several studies draw a distinction between so-called “normal” fluctuations and “crisis” fluctuations, the magnitude of which exceeds a particular threshold (Rancière et al., 2008; Hnatkovska and Loayza, 2005).

The literature on the economic vulnerability of developing countries emphasises the significant contribution made by the magnitude and frequency of external and natural shocks to the structural vulnerability of developing countries (Guillaumont, 2007, 2009a, 2009b, 2010; Cariolle, 2011; Loayza and Raddatz, 2007; Combes and Guillaumont, 2002). Research by Mauro and Becker (2006) identifies the external shocks that cause growth shocks, such as a deterioration in relation to terms of trade and a sudden halt to the movement of capital. Similarly, Raddatz (2007) examines the sources of volatility in GDP in the Least Developed Countries. He sets out an analysis of the total and relative contribution of external shocks to volatility in GDP based on a breakdown of GDP variance. Raddatz shows that external shocks (terms of trade, price of primary products, LIBOR and development aid) have only a marginal effect on the volatility of the growth rate of GDP, whilst internal factors related to economic policy (level of public deficit, inflation and overvaluation of exchange rates) make a significant contribution. These results confirm those of Fatas and Mihov (2006, 2007), showing that economic volatility results in part from volatility in budget policy. Finally, Hnatkovska and Loayza (2005) distinguish between the effect on growth of “normal” fluctuations in GDP (positive or negative, repeated and on an average scale) and the effect of “crises” measured by falls in GDP over a certain threshold. The simultaneous introduction of these two volatility variables in their regressions shows the important and significant effect of a “crisis”
level of volatility on growth and the insignificant effect of “normal” volatility. The same classification of volatility was used by Rancière et al (2008) and applied to the effect of a financial “systemic risk” on growth. They find a positive relationship between financial crisis and growth, which is explained by the leverage effect of companies’ level of indebtedness on their investments.

The literature on macroeconomic volatility therefore covers something of a wide field, showing the significant interest in understanding it and its decisive role on economic performance in relation to growth. As we will see below, however, there is a diverse range of methods available for measuring volatility, although there has been no real discussion of the advantages and disadvantages of such methods since the research carried out by Gelb (1979) and Tsui (1988).

1.2. Measures of economic volatility: a variety of indicators.

Common definitions of volatility often refer to the notion of disequilibrium. Measuring economic volatility involves evaluating the deviation between the values of an economic variable and its equilibrium value. This equilibrium value, or reference value, in turn refers to the existence of a permanent state or trend. In statistical terms, economic volatility is traditionally measured by the second (standard deviation) or sometimes a higher moment1 (Rancière et al, 2008), of the distribution of a variable around its mean or a trend, which then represents the equilibrium value (to which the variable tends to return quickly after deviating in response to a shock). It is frequent for macroeconomic series (GDP, export revenues, final consumption) to be “non-stationary”, i.e. they fluctuate around a trend which itself varies over time, or for shocks to make the variable deviate from its previous tendency over the long term or permanently. It then becomes necessary to use so-called “stationarisation” techniques in order to separate the permanent (or trend) component from the transitory (or residual) component of the evolution of a series (see section 2.1). The volatility indicators obtained using these techniques are intended to reflect the effects of the episodic variations of an economic series around a reference value (mean, deterministic trend or variable over time). Calculating volatility thus relies on two key questions: that of calculating a reference value – or the choice of a stationarisation method – and measuring fluctuations around said reference value. A distinction can be drawn between two main families of volatility indicators: on the one hand, those which measure the variability of an economic series, i.e. taking into account all of the transitory variations of a statistical series, and on the other, those which measure economic uncertainty, or the unpredictability of variations in total variability (Wolf, 2005). We set out below a review of the main indicators of economic volatility as presented in the literature, noting how reference values and fluctuations have been calculated by various authors. A summary table of these indicators can be found in Appendix A.1.

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1 Asymmetry and kurtosis coefficients of financial values are often used as measures of risk in the financial sector. In economics, only Rancière et al (2008) have, to our knowledge, incorporated the effects of economic volatility into their research.
Economic volatility as the standard deviation of the growth rate of a variable

Most of the research proposes measuring volatility on the basis of the standard deviation of the growth rate of a variable, which assumes that said variable is stationary at first difference. In other words, this approach puts forward restrictive hypotheses as to the behaviour of a series without any prior testing.

Ramey and Ramey (1995), for example, propose studying the effect of economic variability using the standard deviation of the growth rate of GDP per-capita. Servén (1997) examines the effects of volatility on investment in sub-Saharan Africa and uses two measures of macroeconomic volatility, namely the standard deviation and coefficient of variation of several aggregates\(^2\) (terms of trade, black-market premium, inflation, etc.). Acemoglu et al (2003) study the effect of institutional quality on macroeconomic volatility and measure the latter using standard deviations of GDP growth rates and terms of trade. Similarly, Di Giovanni and Levchenko (2010), and Van der Ploeg and Poelhekke (2009) examine the effects of a high level of exposure to external shocks and measure macroeconomic volatility using the standard deviation of the growth rates of terms of trade, GDP per inhabitant and exports. Raddatz (2007) also uses measures based on the standard deviation of the growth rate of several macroeconomic variables (price of primary products, terms of trade, aid per inhabitant, GDP per inhabitant and LIBOR) to examine the contribution of external shocks to the volatility of GDP in African countries. It is thus common to apply measures of volatility based on variance (standard deviation or coefficient of variation) to differentiated series such as GDP, terms of trade, export revenues, prices of goods or international interest rates.

Economic volatility as the standard deviation of the residual of an econometric regression

Other measures of volatility are based on the residual or explanatory power of econometric regressions. Pritchett (2000) proposes three measures of volatility. The first is based on the coefficient of determination of a growth-rate regression on a linear temporal trend. The lower the coefficient of determination, the more the explanatory power of the temporal trend is limited and the greater the level of volatility. The second measure is based on the difference in growth rates before and after a break year identified by minimising the sum of the squares of the residuals of a regression on a simple linear trend\(^3\). The author also proposes a measure of economic volatility based on calculating the standard deviation of the residual of a regression of GDP on a mixed deterministic and stochastic trend, along with Servén (1998), Combes and Guillaumont (2002), and Guillaumont and Chauvet (2007)\(^4\). In a similar approach, Lensink and Morrissey (2006) examine the

\(^2\) See section 3.2 for standard deviation and coefficient of variation formulae.

\(^3\) \[ y_1 = a_1 I_1(t < t^*) + b_1 t I_1(t < t^*) + a_{11} I(t > t^*) + b_{11} t I_{11}(t > t^*) + e_t \]

Where \(I()\) is an indicative function, and the break year, \(t^*\), is chosen to minimise the sum of the square of residuals, and \(t^* - t \geq 6\) and \(T - t \geq 6\).

\(^4\) Amongst other things, Servén (1998) uses a specification similar to that used by Combes and Guillaumont (2002), to measure uncertainty, imposing a nullity constraint on the coefficient associated with a second lag.
effect on economic growth of the volatility of Foreign Direct Investments (FDI) as measured by the standard deviation of the residual of a regression of FDI on its three lags and a deterministic trend.

Other authors have tended to concentrate more on the effects of economic uncertainty on investment and growth. Ramey and Ramey (1995) propose a measure of the uncertainty component of volatility based on the standard deviation for prediction error against the growth rate (the predicted value being obtained by a growth rate regression against a quadratic trend, linear trend, two GDP lags and the initial values of the share of investment in GDP, population and human capital). Servén (1998) and Dehn (2000) in turn examined the effect of economic uncertainty on investment using several measures of uncertainty generated by price volatility in raw materials and calculating the conditional standard deviation for prediction error for these series obtained using a GARCH process (1,1).

Volatility measures based on the residual of econometric regressions thus have the merit of being based on a less restrictive formalisation of the process underlying the change in the trend of economic series. Nonetheless, it remains to be seen whether said formalisation allows proper series stationarisation, and whether the interpretation of the residual is correct (uncertainty or variability?).

**Economic volatility as the standard deviation of the cycle isolated by a statistical filter**

Finally, several studies have used the filtered value of a statistical series as a reference value. This technique can be used to disaggregate a series into trend variations (long term) and cyclical variations (short term). This type of volatility indicator is therefore based on cyclical or cycle fluctuations. The filtering technique is different from the previous two methods insofar as it does not formulate the behaviour of a series in advance (order of integration, difference-stationarity or trend-stationarity) and filters series on the basis of their past and future behaviour. Section 2.3 examines this technique in more detail.

Dawe (1996) thus filters export series using a moving average based on five years, i.e. on $[t-2; t+2]$, and bases his volatility measurement on the average difference between the series observed and this moving average. Other authors use the Hodrick-Prescott (Hodrick and Prescott, 1997) filter to calculate their volatility indicator; they include Chauvet and Guillaumont (2007), who use the standard deviation of the development aid cycle isolated using this filter (HP), i.e. the standard deviation of the difference between the value smoothed by the filter and the observed value of their aid variable. Becker and Mauro (2006) propose a shock variable based on the HP filter and identify decreases in GDP as an event equating to a decline in filtered GDP of over 7%. Hnatkovska and Loayza (2005) isolate the cyclical fluctuations in GDP series using the Baxter and King filter (1999) and calculate their standard deviation. Finally, Afonso and Furceri (2010) use the standard deviation of the cyclical component of public spending and tax revenues isolated using HP and Baxter-King filters.

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5The authors regress the following model for the whole of the sample for each country:

$$
\Delta \sigma_a = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 \text{dummy}_{\text{wea}} + \varphi \text{dummy}_{\text{wfa}} + \beta \gamma_{a-1} + \gamma \lambda_{a-2} + \delta \text{pop}_{a0} + \delta \text{Hkm}_{a0} + \theta \text{Khm}_{a0} + \epsilon_a
$$
The usual indicators of economic volatility can therefore be distinguished by the method used to calculate the reference value selected. It is thus possible to draw a distinction between indicators based on first-difference series variance, indicators based on the variance of the residual of a more complex model of economic series, and those based on cyclical variance identified by applying statistical filters. These distinctions become more blurred, however, with regard to the method used to calculate deviations from the reference value, since the literature limits the analysis of volatility to an analysis of the variance of volatility, i.e. the average magnitude of fluctuations\(^6\).

In the following section, we present an analysis of the various methods used for calculating a reference value based on export data for the period 1970-2005 and illustrate the differences in the analysis depending on the method used. In section three, we outline various ways of characterising the fluctuations of a variable around a reference value, showing that it is possible to quantify volatility not only by the average magnitude of economic fluctuations, but also in terms of their asymmetry and the occurrence of extreme variations (or kurtosis).

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2. The question of reference values

If volatility refers to the notion of disequilibrium, then it must be measured using stationary series. Most economic aggregates, however, are not “naturally” stationary. It therefore becomes necessary to calculate a reference value or trend value, around which series will be stationary, which in turn means identifying the right method of stationarisation.

Or \( y_t = \mu_t + \varepsilon_t \), a non-stationary process with \( \mu_t \) a non-constant term and \( \varepsilon_t \) the residual. Stationarising \( y_t \) consists of calculating or estimating the trend component \( \mu_t \) so that the residual (or cycle) \( \varepsilon_t \) meets the following conditions\(^7\):

\[
\begin{align*}
\text{(1) } & \quad E(\varepsilon_t) = 0; \\
\text{(2) } & \quad V(\varepsilon_t) = \sigma^2 < \infty \text{ for all of } t; \text{ and} \\
\text{(3) } & \quad \text{Cov}(\varepsilon_t; \varepsilon_{t-k}) = \rho_k.
\end{align*}
\]

These three conditions require the residual \( \varepsilon_t \) to have a zero mean (1), a variance (2) and a finite autocovariance (3) independent of time. Volatility measures are based on the residual (or cycle) \( \varepsilon_t \), which therefore reflects only transitory fluctuations. If the series is poorly stationarised, variations which are attributable to a long-term (or permanent) change in \( y_t \) may be included in the residual, thus breaching conditions (1), (2), and (3). The volatility measures based on them would therefore be incorrect, because they do not correspond with the definition given to them. Stationarisation is a prerequisite condition for calculating volatility based on non-stationary level series. Calculating a reference value is therefore a fundamental step, since it results in identifying and isolating the trend or permanent component in the change of an economic variable, from its transitory or stationary component (Dehn, 2000; Hnatkovska, 2005). To understand this issue in more detail, we first examine the theoretical breakdown of change in statistical series. We then present the principles and properties of the usual methods for calculating reference values, which we apply to export data.

We illustrate our analysis using the annual change in export revenues for 134 (developed and developing) countries over the period 1970-2005 from the World Development Indicators\(^8\). The advantage of using these series is that export volatility is an important aspect of macroeconomic volatility, which is addressed in depth in the literature on economic volatility. Fluctuations in export revenues may reflect both the change in domestic (changes in domestic production conditions, natural disasters, etc.) and international economic conditions (volatility in global prices). Within this framework, we present standard techniques for series stationarisation and calculating the volatility of exports applicable to a broad range of developed and developing countries over the period 1970-2005.

\(^7\)We are referring here to conditions of “weak” stationarity.

2.1. Breakdown of economic series

Observation of series behaviour

Figure 1 shows the changes in export revenues in constant dollars (2000) in six different countries (South Korea, Argentina, Venezuela, Kenya, Ivory Coast and Burundi) and the spectrum densities associated with them. The spectrum of a series is a representation of the contributions of each frequency variation9 to the total variation of the series. Observing the spectrum density of a series then makes it possible to identify whether the change in a series is dominated by variations over a longer period or shorter period. Examining the spectrum is therefore a very useful diagnostic tool if we wish to represent correctly the dynamics of change in a series. A peak at a given frequency indicates that a significant proportion of the total variance in a series can be explained by the variations in said frequency.

Figure 1 shows that the countries represented had a change in exports dominated by variations over a long period, with an increasing trend (excluding Burundi) and a decreasing spectrum density. Except for South Korea10, a large proportion of the series spectrum is located around variations in periodicity of around 20 years (with a peak in density at a frequency of around 0.05). It all appears that variations in average periodicity contribute strongly to total variability for these sample countries. In particular, variations between 5 and 7 years (with a frequency between 0.1 and 0.3) seem to contribute substantially to total variability in series from Venezuela, Ivory Coast and Burundi. This reflects the conclusions reached by Rand and Tarp (2002), and Aguiar and Gopinath (2007), according to whom developing economies experience greater volatility in their rate of economic growth than developed countries. Finally, an examination of the spectrum for the change of exports can be used to identify the existence of peaks of density at high frequencies corresponding to periodicities of around 2-3 years, which suggest that the total variability of the series used can also be explained by fluctuations over short periods.

Examining the changes in export series for a sample of countries therefore suggests that, although variations in export series over a long period explain most of their variability, fluctuations over a medium period also play a part. The choice of a reference value is therefore important, since this makes it possible to distinguish trend variations (long/medium periodicity) from the short-term transitory variations on which volatility calculations are based. A subsequent theoretical breakdown of economic series provides additional information for understanding changes in statistical series.

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9The calculation for switching from frequency to period is as follows: F=1/T, where F = frequency and T = period.

10South Korea presents a “Granger profile” (Granger, 1966), with most of the power of the spectrum close to a zero frequency.
Theoretical breakdown of series

According to Dehn (2000) and Hnatkovska (2005), economic series have a \textit{trend} or \textit{permanent} \((y^p_t)\) component and a \textit{cyclical} or \textit{transitory} \((y^c_t)\) component:
\[ y_t = y^{p_t} + y^{c_t} \quad (1) \]

The *permanent* component is made up of a *deterministic* part \((y_0 + \alpha t)\), with a temporal trend \(t^{11}\), and a *stochastic* \((\varepsilon^{p_t})\) part:

\[ y^{p_t} = y_0 + \alpha t + \varepsilon^{p_t} \quad (2) \]

\(\varepsilon^{p_t}\) represent stochastic shocks affecting the series trend on a *permanent* or *prolonged* basis. By way of example, productivity shocks or a change in the preferences of economic agents can affect the change in an economic series over the long term.

The *cyclical* or *transitory* component comprises a *predictable* \((y^{CT}t)\) component – associated with structural factors such as the level of development, foreign exchange system, the size of the country, etc.) and an *unpredictable* \((\varepsilon^{c_t})\) component:

\[ y^{c_t} = y^{CT}t + \varepsilon^{c_t} \quad (3) \]

\(\varepsilon^{c_t}\) represents unpredictable shocks with a temporary effect on the series cycle, such as sudden changes in international prices of raw materials or unforeseen climatic events.

**Volatility as a measure of variability, risk or uncertainty?**

As Azeinman and Pinto (2005) suggest, the literature generally sees volatility as associated with economic risk or uncertainty\(^{12}\). According to the authors, whilst volatility provides information on the observed results of a variable, it can also, by extension, provide information on possible results and thus represent an approximation of the risk associated with it. The volatility indicators presented in section 1.2 would then be similar to risk indicators. The authors emphasise, however, that such a measure can overestimate risk by also including predictable fluctuations. Pure risk or uncertainty would then need to be measured by the residual obtained from a volatility prediction model, e.g. conditional variance models such as GARCH models (Dehn et al, 2005; Dehn, 2000; Serven, 1998). Techniques for measuring volatility would therefore be divided into two main families, namely those which provide *measures of the total variability* of a series, and those which provide *measures of uncertainty or risk* (Wolf, 2005).

Models based on uncertainty indicators, however, such as GARCH models, generally apply to high-frequency economic data (daily, monthly or quarterly price changes, for example), whereas here we are examining volatility in exports, which are reported annually\(^{13}\). As a result, this paper does not address uncertainty indicators but instead reviews calculation methods for the reference values used as the basis for indicators of volatility in terms of variability, i.e. based on the variations of \(y^{c}\) in (3).

\(^{11}\)Here we are examining the case of a linear deterministic trend, accepting that this may take diverse forms (quadratic and exponential trends, etc.)

\(^{12}\)Ignoring the “Knightian” distinction between risk and uncertainty.

\(^{13}\)As in the majority of research on macroeconomic volatility.
2.2. Parametric approach

Most measures of macroeconomic volatility are based on a univariate parametric approach, which models economic series on the basis of past change over a given period. Much of the research set out in the previous section uses volatility indicators based on residual variance. The most common techniques are presented below.

Estimate based on a linear deterministic trend

The traditional approach consists of developing a volatility indicator based on an average deviation around a linear trend. In reality, export series, like all other actual macroeconomic variables (GDP, exports, interest rates, etc.) are series dominated by low frequencies\(^{14}\) (figure 1), which justifies modelling them using a deterministic trend (linear, polynomial or exponential). In its simplified (linear) form, this technique consists of estimating the following model:

\[
y_t = \alpha + \beta t + \varepsilon_t
\]  

(4)

Where \(y_t\) is the variable whose volatility is being measured, \(\alpha\) a constant, \(t\) a linear trend, and \(\varepsilon_t\) a zero mean error term. In this case, the reference value is the trend:

\[
\hat{y}_t = \hat{\alpha} + \hat{\beta} t
\]  

(5)

Deviations from the trend (\(\varepsilon_t\)) in principle have no permanent effects on \(y_t\). In other terms, these deviations are assumed to be stationary around the trend and can therefore represent the volatility of \(y_t\). A measure of volatility based on \(\varepsilon_t\) thus relies on three key hypotheses: i) that the series changes at a constant rate over time, ii) that the long-term change in the series is perfectly predictable and iii) that all deviations or shocks affecting it are transitory around the trend. Beveridge and Nelson (1981) highlighted the limitations of an approach of this kind.

\(^{14}\)A series whose variations over a long period are those which contribute the most to total variance. See Guay and Saint-Amant (1997).
To illustrate these limitations Figure 2 shows the actual change in exports and their trend as shown in (7) in Belize between 1980 and 2004, and in Argentina between 1970 and 2005 (Figure 3 shows the residuals for the estimates produced). Although the change in export revenues in Belize appears at first sight to be linear, the series observed may, in spite of this, depart from its trend over the long term. This problem is all the more obvious in Argentina, where the growth in export revenues does not, at first sight, follow a linear trend. This example illustrates the limitations of this technique for calculating volatility. Such a specification risks overestimating the importance of shocks by wrongly including a part of the non-constant trend in the residual.
Estimate based on a mixed trend

Up to this point, we have assumed that deviations from the deterministic trend are only transitory. Equation (2), however, suggests that it is possible that residual variations ($\varepsilon^T$) are not transitory, and have a permanent effect on the trend followed by a series. It thus becomes possible to estimate the reference value on the basis of a stochastic trend (a random process below), represented by the following first-order autoregressive AR(1) process,

$$y_t = y_{t-1} + \varepsilon_t, \quad \text{avec } \varepsilon_t \sim N(0, \sigma^2) \quad (6)$$

which we can re-write as follows:

$$y_t = y_0 + \sum_{i=1}^{t} \varepsilon_i$$

In this case, the change in $y_t$ is determined by a successive history of random shocks, with the result that a shock occurring in the past, even the distant past, has the same effect on the series as a shock in the present (long-memory process). The series is then considered to be difference-stationary or that its trend follows a random path. The series contains a unit root. After differencing, the series can be rewritten as follows:

$$\Delta y_t = \varepsilon_t, \quad \text{avec } \varepsilon_t \sim N(0, \sigma^2)$$

This hypothesis is nevertheless quite strong in the context of examining macroeconomic variables dominated by low frequencies (Nelson and Kang, 1981). The analysis of the export series and their spectrum in section 2.1 (see Figure 1) would justify specifying a mixed trend, suggesting the existence of fluctuations around a trend which is both deterministic and stochastic. Namely the process AR(1), which also includes a deterministic trend:

$$y_t = \alpha + \beta t + \delta y_{t-1} + \varepsilon_t \quad (7)$$

Our aim is to discover whether the trend as specified in equation (7) makes it possible to stationarise the export series for our sample of countries correctly. To do so, we calculate the p-value of the unit root test using Maddala-Wu panel data (based on the Phillips Perron test), carried out in 134 countries, and Fisher statistics on the joint nullity hypothesis for coefficients associated with drift, trend and lag (table 1).
Table 1: Specification and unit root test on panel data

<table>
<thead>
<tr>
<th>H₀: the series is non-stationary</th>
<th>Prob&gt;Chi²</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Δy_{it} = α_i + β_t t + φ_1 y_{i,t-1} + ε_{it}$</td>
<td>1.000</td>
<td>47.47</td>
</tr>
<tr>
<td>$ΔΔy_{it} = α_i + β_t t + φ_2 Δy_{i,t-1} + ε_{it}$</td>
<td>0.000</td>
<td>36.61</td>
</tr>
</tbody>
</table>

Countries (Observations): 134(3693).

The results of the tests carried out thus do not justify the rejection of a null hypothesis for unit root and tend to justify the use of a mixed trend with drift (cf. equation (9)). The non-stationarity hypothesis is thus rejected once the series is differenced for a second time, and the F-test statistics lead us to reject the joint nullity hypothesis for the specification coefficients (9). Figure 4 illustrates the trends obtained for the case of Belize and Argentina.

Figure 4. Change in export revenues, mixed trend.

![Figure 4](image)

Figure 5 shows the correlogram for the residuals of equation (7) estimated for Belize and Argentina respectively. An examination of the correlogram shows white noise, because the correlations associated with lags are not significantly different from zero. With regard to these two example countries and those shown in Appendix A.2, using a mixed trend therefore proves to be more appropriate to stationarise export revenues.

However, an examination of Figure 4 and Appendix A.2 suggests that this method of estimating creates a trend whose profile appears to be a slightly smoothed and offset version of the change seen in the exports. This trend therefore seems to reproduce in $t$ the change in exports observed in $t-1$, which therefore contributes to an artificial creation of volatility. This phenomenon is all the more marked in the cases of Argentina (Figure 4), Venezuela, Burundi and Kenya (Appendix A.2).

---

15The Portmanteau statistics, which are not presented in this article, do not make it possible to reject a null hypothesis for a white-noise process. They can be supplied to the reader on request.
Estimate based on a rolling mixed trend

The main problem with a mixed trend as calculated in the previous sub-section is that it is based on a strong hypothesis of constancy over time of the coefficients associated with the trend of the series. Effectively, this so-called “global” trend is predicted each year for each country based on coefficients estimated for the whole period of data availability. It also excludes the possibility of a change of regime in the deterministic and stochastic change of the series concerned. Maddala and Kim (1996) emphasise that important changes in the deterministic component of the trend taken by a series can lead, wrongly, to a failure to reject the unit root hypothesis. It is therefore possible that the mixed trend we have estimated attributes to random trend variations a change in the deterministic change of the series, which may then lead to an overestimate of the magnitude of fluctuations. Although tests do exist (e.g. CUSUM, Max Chow) to identify breaks in a trend during unit root tests (Maddala and Kim, 1996), an alternative and practical solution is to produce a “rolling” estimate of the mixed trend over a shorter period (Guillaumont, 2007), allowing the coefficients estimated to change from year to year and thus reflecting recent changes in the series trend. This “rolling mixed trend” is calculated for each country and each year based on estimating equation (9) over the period \([t; t-k]\), rather than over the whole of the period:

\[
\hat{y}_t = \hat{\alpha}^T + \hat{\beta}^T t + \hat{\delta}^T y_{t-1}
\]

and

\[
y_t = \hat{y}_t + \epsilon_t
\]

where \(T\) is the estimation period for the trend corresponding to \((t; t-k)\).

The results obtained when \(k = 12\) are shown for Argentina in Figure 6 and compared with those obtained based on the previous “global” mixed trend. Appendix A.2 shows the predictions and
correlograms for residuals obtained using global and rolling mixed trends for other country examples. We can see that the trend thus obtained is smoother than with the previous model and that it does not have a “sawtooth” profile resulting from constant parameters over time. Similarly, an examination of the correlogram in Figure 6 and Figure 7, and the correlograms in the appendix (Appendix A.2), suggests that the residuals resulting from this approach are stationary for the country examples chosen.

Nonetheless, this technique presents a number of disadvantages. On the one hand, it reduces the time coverage of volatility indicators, since the first trend value is only available from \( t = 1+k \). On the other hand, a bias may appear as the result of a limited trend estimation period.

Estimates based on a rolling mixed trend are not necessarily an ideal solution where there is a break in the trend of the series. Changes prior to the break may continue to exhibit inertia when the rolling trend is estimated, once the break has occurred. Moreover, by proposing an estimate of the trend over a shorter period than the global mixed trend, this technique may include some long-periodicity fluctuations in the residual as a result of being more sensitive to medium-periodicity fluctuations. This phenomenon is illustrated in particular by the respective behaviour of two mixed trends applied to Burundi between 1985 and 1995 in Figure 7. The rolling mixed trend tends to underestimate the trend for exports to fall relative to the “global” trend over the same period.

The choice of period for calculating the trend is therefore important. In this case we have chosen an estimation period for the rolling trend of 12 years, in order to highlight the advantages and disadvantages of this technique compared with the global mixed trend. It could be considered that a rolling trend calculated over a period of between 10 and 20 years represents a reasonable basis with regard to the spectrum densities presented previously in Figure 1, since medium-term fluctuations seem to contribute strongly to the total variability of the series. This choice should be justified based on a prior examination of series behaviour (unit root tests, graphical examination of series and examination of their spectrum density) and supported by a study of the literature on the topic.
2.3. Filtering approach

Some research on economic volatility uses a statistical filtering approach to isolate the cyclical and trend components of changes in the series (Becker and Mauro, 2006; Chauvet and Guillaumont, 2007). The standard deviation of the cyclical component then becomes an example of a volatility indicator. The most popular filter remains the Hodrick-Prescott filter (1997). This can stationarise potentially integrated series up to order four (King and Rebelo, 1993). The band-pass (BP) filter put forward by Baxter and King (1999) is also used in the literature on economic fluctuations (Hnatkovska and Loayza, 2005). Although the BP filter maintains the properties of the series more accurately, this advantage comes at the price of a loss of observations at the end of the sample\textsuperscript{16}. Moreover, when applied to our export data, both techniques give extremely similar results\textsuperscript{17}. In this article we therefore only present the results obtained with the Hodrick-Prescott (HP) filter.

The advantage of the filtering method compared with those described above is that it does not impose \textit{a priori} any particular (and sometimes arbitrary) form (mixed trend, deterministic, random, etc.) on the behaviour of the series. Moreover, a statistical filtering method enables changes to the trend over time, which is a definite advantage over the time series-based approach we have

\textsuperscript{16}The BP filter is a bilateral filter, which requires a minimum number of observations before and after each filtered observation point in order to increase the precision of the filtering. Baxter and King recommend ignoring the first 12 and last 12 quarters when filtering quarterly series. For annual series, they recommend ignoring the first three and last three years of the sample.

\textsuperscript{17}Available to the reader on request.
presented. Hodrick and Prescott break down the change in a series into a non-stationary trend component \((y_t^T)\), and a stationary cyclical component \((y_t^C)\): 
\[ y_t = y_t^T + y_t^C, \quad T = 1, 2, 3, \ldots, t. \] 
(9)

The HP filter consists of isolating the cyclical component by optimising the following programme in relation to \(Y_t^p\):
\[ \min \left\{ \sum_{t=1}^{T} (Y_t - Y_t^p)^2 + \lambda \sum_{t=2}^{T-1} (\Delta^2 Y_t^p)^2 \right\} \] 
(10)

It is similar to a symmetrical infinite moving average filter. The parameter \(\lambda\) is called a “smoothing parameter”. The first term of the equation (10) minimises variance in the cyclical component \((Y_t^C)\) whilst the second term smoothes the change in the trend component. When \(\lambda\) tends towards infinity, the variance in the growth of the trend component tends towards 0, which implies that the trend component – or filtered series – is close to a simple linear temporal trend. Conversely, when \(\lambda\) tends toward 0, the filtered series is close to the original series. Isolated cyclical fluctuations will thus exhibit a higher (short) frequency (periodicity) the lower the value of the smoothing parameter.

The choice of the value of \(\lambda\) remains arbitrary and there is as yet no unanimity in the literature. Whilst Hodrick and Prescott advocate a parameter \(\lambda\) equal to 100 for annual data, some studies suggest a higher value, of between 100 and 400 (Baxter and King, 1999), whilst others prefer significantly lower values, of between 6 and 10 (Maravall and Del Rio, 2001). Given that the aim of this article is not to debate the appropriate value for the smoothing parameter, we will compare the results obtained when it is equal to 100 and 6.5. We will thus obtain deviations compared with a long-term trend \((\lambda=100)\) and a medium-term trend \((\lambda=6.5)\).

An examination of Figure 8 suggests that use of the HP filter can apply to a large number of countries with varied profiles for changes in exports. As predicted, values filtered with a smoothing parameter of 6.5 are more sensitive to fluctuations in the medium term than those filtered with a parameter of 100. Appendix A.3 shows the correlograms between the cycles of the countries shown in Figure 8. Both these figures thus suggest that the cycles extracted are stationary and not significantly autocorrelated. The p-values of the Phillips-Perron test below the correlograms in Appendix A.3 show that we can reject the null hypothesis for first-order integration of the cycle with a confidence interval of 99%, which indicates that the series seems to be correctly stationarised for these countries with this technique.

The approach nonetheless presents a number of disadvantages. Whilst this method does not impose a particular functional form on the change in the trend, it does impose, in an \textit{ad hoc} manner, weightings identical to the values before and after the filtered value. It thus suggests the hypothesis that the cyclical component and trend component of economic series are independent. In light of the significant amount of research on the effects of macroeconomic volatility on long-term growth, however, this hypothesis seems restrictive. Moreover, a further defect of the HP filter is that it becomes unilateral at the end of the sample, thus giving less good results at the start and end of the
data availability period (Van Norden, 2004). Finally and above all, as regards the choice of a smoothing parameter, a low value provokes compression effects: part of the short-periodicity cyclical variation can be attributed to the trend. The estimated trend then appears to be highly volatile and the cyclical component – and therefore volatility – may be underestimated in this case. Conversely, the choice of a high smoothing parameter provokes leakage effects: some of the long-periodicity variations can be attributed to the cyclical component. The trend thus appears less volatile and the cyclical component tends to be overestimated. As regards macroeconomic volatility in developing countries, an examination of the spectra of the export series in Figure 1 suggests the choice of a low smoothing parameter, given the significance of medium-term fluctuations. Figure 8 shows a graphical representation of the consequences of choosing a high or low smoothing parameter.

As we have seen in this section, the method of calculation used for the reference value is central to calculating volatility. If we wish to reflect the effects of transitory variations – for example due to economic cycles, certain commercial shocks or climatic events – it is essential to use an adequate series stationarisation method. In this regard we have set out the two main approaches adopted in the literature on macroeconomic volatility: the parametric approach and the filtering approach. Although both these approaches provide acceptable results in relation to stationarisation, the global mixed trend seems to create volatility artificially: this is less true of the rolling mixed trend and the filtering method.
Figure 8. Export revenues smoothed by HP filter

Argentina

Venezuela

South Korea

Ivory Coast

Kenya

Belize
3. Calculating deviation from the trend

Unlike shock variables, which reflect instantaneous fluctuations, measuring volatility involves calculating a deviation which reflects an accumulation of fluctuations. The deviations presented in this section consist of summing the deviations around a trend over a given period, so as to highlight a particular aspect of the distribution of a variable (stationary or stationarised): its variance, asymmetry or degree of kurtosis (or flattening) of the distribution.

Limiting the analysis of volatility only to traditional indicators based on the variance of a variable, such as standard deviation or the coefficient of variation\(^{18}\), can mask other important aspects of volatility, such as the asymmetry of deviations and the occurrence of extreme deviations. In fact, measures such as the standard deviation of deviations, which measure the average magnitude of a distribution, do not take account of the asymmetry of the reactions of economic agents to positive or negative shocks. Similarly, they are not able to identify whether the distribution around the trend is characterised by frequent shocks on a limited scale, or dominated by infrequent shocks on a large scale. These characteristics of a distribution are important, however, if, for example, the aim is to study how volatility affects the behaviour of economic agents. We present below the possible options for quantifying these phenomena, based on the various moments of distributions around the reference values presented in the previous section.

3.1. Calculation period for volatility

If an indicator of volatility is intended to reflect past experience of shocks, the choice of period over which the deviation is calculated is a first step towards calculating said deviation. If, for example, the aim is to examine the short-term effects of volatility, it is common to calculate an indicator reflecting fluctuations over the last five or ten years. If the aim is to study the medium/long-term effects of volatility, a calculation period of over ten years may be needed. Moreover, where the change in a series presents distinct episodes of volatility over time, it may be relevant to align the period over which deviations are summed to the approximate period of the episodes of volatility (Wolf, 2005). In fact it is possible to sum over a short period, deviations around a trend which has itself been estimated over a longer period (see section 2.2). In this case, it will be a matter of studying the short-term effects of fluctuations around a shorter- or longer-term trend.

In the remainder of the document, we compare the results from different types of volatility indicators used to analyse the three dimensions (or moments) of the distribution of a variable: the scale of values of a variable, their asymmetry and the occurrence of extreme deviations.

\(^{18}\)Their respective formulae are set out at the end of this section.
3.2. The magnitude of volatility

Numerous research articles cited in the list of references examine the consequences and determinants of the magnitude of volatility. Several methods are available to quantify this aspect of volatility. The most common method consists of calculating the standard deviation of a variable based on its reference value:

\[
INS_1 = 100 \times \sqrt{\frac{1}{T} \sum_{t} \left( \frac{y_t - \hat{y}_t}{\hat{y}_t} \right)^2}
\]

with \( T=1, \ldots, t \).

We compare the deviations with the reference value in order to ensure comparability between the indicators of volatility of a variable whose order of magnitude differs from one country to another. It is also possible to calculate the average absolute deviation of deviations from the trend:

\[
INS_2 = 100 \times \frac{1}{T} \sum_{t} \left| \frac{y_t - \hat{y}_t}{\hat{y}_t} \right|
\]

with \( T=1, \ldots, t \).

Where \( y_t \) is the observed value of the series, and \( \hat{y}_t \) is the reference value. The question of the respective advantages of standard deviation and absolute average to measure the ‘average deviation’ of a distribution is an old debate (Gorard, 2005). The preference for standard deviation is historic and is related in part to the fact that numerous statistical tests and numerous indicators are based on it. The preference arose because i) it was shown that in the case of a normal/Gaussian distribution, standard deviation provides a more efficient estimate of ‘average deviation’ than average absolute deviation; and ii) it is easier to manipulate in algebraic terms. In the context of an economic analysis of the effects of the magnitude of volatility, the use of standard deviation is justified in the case where the effects of volatility increase exponentially with the size of fluctuations. It has been shown, however, that average absolute deviation is a more efficient measure where there are measuring errors, or when the distribution is not normal. We illustrate the differences between these two measures of the magnitude of volatility based on the results obtained for Venezuela and Kenya shown in Figure 9. Both of these measures are calculated on a rolling annual basis over a calculation period of five years (t; t-5). Figure 9 suggests that the peaks of volatility corresponding to the end of the 1970s, 1980s and the early 1990s and 2000s are more heavily weighted when we use the mean square deviation (\( INS_1 \)). We subsequently prioritise use of the standard deviation (INS1) as a measure of the average magnitude of deviations, given that this method of calculation makes it possible to distinguish more easily between different episodes of volatility, because of its frequent use in the literature and because the other measures of volatility presented in the paper are based on this statistic. We then compare the standard deviations obtained from the reference values presented in the previous section. This time, deviations are calculated over the period 1982-2005. The results are shown in tables 2 and 3, and in figure 10.
Moreover, it appears that measures of the magnitude of volatility calculated on the basis of mixed trends – global and rolling – and the HP filter are very similar, as suggested by the correlations shown in table 2 and figure 10. Nonetheless, the magnitude of volatility around the global mixed trend and the HP100 filter tends on average to be higher than other indicators.

In the same way that using an HP filter with a high smoothing parameter can provoke leakage effects and result in an overestimate of the magnitude of volatility, a mixed trend may also wrongly include elements of the trend component in the residual (see sections 2.2 and 2.3). Moreover, a trend estimated over a long period is less influenced by medium-term variations than a mixed trend estimated over a shorter period (see, for example, Appendix A.2. Finally, the propensity of this technique to create volatility artificially, as we saw in the previous section, contributes to an average increase in volatility scores.

Table 2. Correlation between magnitude of volatility indicators.

<table>
<thead>
<tr>
<th></th>
<th>(1) Global mixed trend</th>
<th>(2) Rolling mixed trend</th>
<th>(3) HP 6.5</th>
<th>(4) HP 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td>0.92*</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>0.96*</td>
<td></td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>0.87*</td>
<td>0.80*</td>
<td>0.87*</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* Significant at 5%. Observations: 134.
Table 3. Descriptive statistics of magnitude of volatility indicators (%).

<table>
<thead>
<tr>
<th></th>
<th>Volatility Mixed trend</th>
<th>Volatility Rolling Mixed trend</th>
<th>Volatility HP 6.5</th>
<th>Volatility HP 100</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>13.6</td>
<td>11.5</td>
<td>9.2</td>
<td>13.3</td>
</tr>
<tr>
<td><strong>Standard</strong></td>
<td>8.7</td>
<td>7.8</td>
<td>6.1</td>
<td>7.8</td>
</tr>
</tbody>
</table>

Observations: 134.

The values smoothed by the HP filter are calculated on the basis of a symmetric moving-average process and used to take account of any breaks in the trend. It has been noted that the trend in the numerous developing countries which make up our sample is more volatile (Rand and Tarp, 2002). This is why the use of the HP(100) filter is relevant if the aim is to reflect the trend in exports in industrialised economies, over a longer period and on a larger scale. As with the global mixed trend, the HP100 filter is therefore less sensitive than other trends to variations over a medium period. This explains why volatility calculated on the basis of the HP100 filter correlates less closely – though still at a high level of around 80% – and shows a higher mean than other methods.

Figure 10. Graphical illustration of correlations between magnitude of volatility indicators.
3.3. The asymmetry of volatility

Measures of magnitude do not provide a means of identifying the effects of the asymmetry of shocks (Wolf, 2005). A study of the effects of the asymmetry of fluctuations is justified, however, in light of the distinction in agents’ responses to negative or positive shocks (Dercon, 2002; Elbers et al, 2007). The coefficient of asymmetry or skewness, describes the macroeconomic volatility profile facing a country, since it indicates the propensity of countries to suffer negative or positive shocks. This value is calculated as follows:

\[
\text{Skewness} = 100 \times \frac{\frac{1}{T} \sum (\frac{y_t - \hat{y}_t}{\hat{y}_t})^3}{\left(\frac{1}{T} \sum (\frac{y_t - \hat{y}_t}{\hat{y}_t})^2\right)^{3/2}} \quad \text{avec} \quad T = 1, \ldots, t
\]

A negative skewness indicates that the distribution is weighted towards the left of the reference value, whilst a positive coefficient indicates a distribution weighted towards the right (see figure 11). A symmetrical distribution will have a coefficient whose value is close to zero. A distribution with a positive (negative) skewness therefore indicates an experience of volatility dominated by positive (negative) shocks. Moreover, asymmetry increases as shocks (whether positive or negative) become greater. In other words, skewness also indicates the propensity of a variable to undergo crises (when negative) or booms (when positive) (Rancière et al, 2008). Although the measures of the magnitude of deviations around a trend presented previously are strongly correlated (see table 2 and figure 10), the asymmetry of distributions of deviations around them can present a significantly different profile depending on the reference value chosen (see Table 4). Thus, for a given magnitude of volatility, the experience of shocks of two different countries can present a contrasting asymmetry (see figure 11).

---

**Figure 11** Graphical illustration of a positive and negative asymmetric distribution of identical magnitude\(^{19}\).

---

Table 4 shows that the correlations between skewness based on different values are weak, which suggests that each of the reference values expresses the asymmetry of shocks differently. The similarity in the profile of distributions of deviations around the filtered values is confirmed by a relatively strong correlation between their skewness (65%), whilst the skewness obtained from the global mixed trend is weakly correlated to the others. The asymmetry of the distribution of results around the mixed trend shows correlations with other, weaker indicators (14% with the skewness-HP(6.5), and 2% with the skewness-HP(100)).

Table 4. Correlations of coefficients of asymmetry calculated over the period 1982-2005.

<table>
<thead>
<tr>
<th></th>
<th>(1) skewness (Global mixed trend)</th>
<th>(2) skewness (Rolling mixed trend)</th>
<th>(3) skewness (HP(6.5))</th>
<th>(4) skewness (HP(100))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>0.23*</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>0.08*</td>
<td>0.14*</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>0.29*</td>
<td>0.02</td>
<td>0.65*</td>
<td>1</td>
</tr>
</tbody>
</table>

* Significant at 10%. Sample = 134 countries.

Figure 12 compares indicators of the magnitude of volatility with indicators of the asymmetry of volatility for each reference value. The straight-line correlations obtained for each reference value suggest a fairly weak positive correlation, particularly if certain outliers are excluded. This graph underlines the fact that for episodes of volatility on a similar magnitude, the asymmetry of shocks may be opposite, suggesting an entirely distinct experience of shock.

The asymmetry of distributions around the reference value is therefore a separate aspect of volatility, which cannot only be assessed by the indicators of magnitude presented in section 3.2.
Figure 12. Correlation between magnitude and asymmetry of volatility indicators, by reference value.
3.4. Frequency of extreme deviations

A final dimension of the volatility of a macroeconomic variable is the occurrence of extreme deviations within a given distribution. This is measured by the fourth moment of the distribution of observations around their reference value, namely kurtosis. This value is calculated as a percentage of the reference value, as follows:

\[
Kurtosis = 100 \times \frac{1}{T} \frac{\sum (y_t - \hat{y}_t)^4}{\left( \frac{1}{T} \sum (y_t - \hat{y}_t)^2 \right)^2} \quad \text{avec} \quad T = 1, ..., t
\]

In the case of a normal distribution, kurtosis is equal to 3 (or 300% when expressed as a percentage of the trend). The kurtosis is a measure of both the peakedness and tails’ fatness of a random variable’s probability distribution relative to those of a normal distribution. It indicates the extent to which the number of observations close to the mean is high compared with observations away from the mean. In other words, a high value indicates that the distribution tends to be pointed around the mean with thick tails. A low coefficient indicates that the distribution tends to be concentrated around the mean with thin tails. Figure 13 illustrates three types of flattening of distributions – leptokurtic (kurtosis > 3), normal (kurtosis = 3), and platikurtic (kurtosis < 3). This thus gives an indication of the propensity of a variable to take high values. The risk of kurtosis is seen as a major risk in finance and on this basis is used as a measure of volatility in its own right. In the case of export volatility, it makes it possible to have an idea of the propensity of certain countries to experience significant variations in foreign trade in terms of volume or prices. As underlined by Rancière et al (2008), a high value of kurtosis should be interpreted with caution since it may both “be generated either by extreme events or by a cluster of observations around the mean that affect the peakedness of the distribution” (p.386). They find that about one-fifth of their sample of countries exhibiting high kurtosis was affected by observations in the neighbourhood of the distribution centre. One should hence look carefully at country data and combine such measure of volatility with a measure of skewness, to ensure that countries actually experienced sharp unusual fluctuations (see figure 15). Figure 15 also illustrates that such combination can reflect country’s propensity to experience...
negative or positive external shocks and thus avoid establishing arbitrary thresholds beyond which countries experience either a “boom” or a “crisis” (Rancière et al, 2008).

Table 5 shows the descriptive statistics for kurtosis associated with each reference value. Distributions around reference values are thus, on average, slightly leptokurtic (higher than 300%) but remain close to the kurtosis value of a normal law. It should be noted that the distributions of deviations around HP(100) filtered values have a similar degree of flattening, on average, to those of deviations around HP(6.5) filtered values. Distributions around mixed trends show, on average, more extreme results than distributions around filtered values.

Table 5. Descriptive statistics for kurtosis calculated for the period 1982-2005.

<table>
<thead>
<tr>
<th></th>
<th>Kurt. (global mixed trend)</th>
<th>Kurt. (rolling mixed trend)</th>
<th>Kurt. (HP(6.5))</th>
<th>Kurt. (HP(100))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean (%)</strong></td>
<td>352.7</td>
<td>367.9</td>
<td>320.0</td>
<td>312.2</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>155.7</td>
<td>173.0</td>
<td>139.0</td>
<td>146.2</td>
</tr>
</tbody>
</table>

Total sample: 134 countries.

Table 6 shows the correlations between kurtoses obtained by the four reference values. Compared with the correlations between dissymmetry coefficients, the correlations between kurtoses are slightly stronger. The degree of occurrence of large-scale positive or negative deviations is logically less influenced by the choice of reference value than their asymmetry. The correlations between the kurtosis of distributions around a rolling mixed trend and those around filtered values, however, are the weakest. The reference values shown in this paper point to different analyses with regard to the propensity of countries to experience extreme deviations.

Table 6. Correlations in kurtosis calculated over the period 1982-2005

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>0.39*</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>0.38*</td>
<td>0.28*</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>0.49*</td>
<td>0.22*</td>
<td>0.62*</td>
<td>1</td>
</tr>
</tbody>
</table>

*Significant at 5%. Sample = 134 countries.

Figure 14 compares indicators of the magnitude of volatility with indicators of the kurtosis of volatility and shows a slightly stronger correlation between these two than between indicators of the magnitude and asymmetry of volatility. The average magnitude of deviations thus seems to be relatively independent of the occurrence of extreme values. By way of example, a country such as the Dominican Republic (abbreviated to “DOM” in the graphs in Figure 14) has a very high kurtosis coefficient but is subject to volatility on a moderate scale. Conversely, Guinea-Bissau (“GNB” in
Figure 14) is a country which, according to the indicators shown here, has experienced volatility on a fairly large scale but has a more platikurtic distribution of observations around the reference values. The strongest positive correlation between the average magnitude of deviations and their kurtosis around the HP(100) filter confirms the propensity that this reference value has to isolate cycles over a longer period, thus explaining a relatively more pointed distribution around the reference value.

**Figure 14. Graphical illustration of correlations between magnitude and kurtosis of volatility indicators, by reference value**
Table 7. Correlations between kurtosis and coefficient of asymmetry for each reference value, 1982-2005.

<table>
<thead>
<tr>
<th></th>
<th>Global mixed trend</th>
<th>Rolling mixed trend</th>
<th>HP (6.5)</th>
<th>HP (100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total sample</td>
<td>+0.65*</td>
<td>+0.51*</td>
<td>+0.12*</td>
<td>+0.35*</td>
</tr>
<tr>
<td>Positive asymmetry</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC&gt;0%</td>
<td>+0.85*</td>
<td>+0.83*</td>
<td>+0.84*</td>
<td>+0.91*</td>
</tr>
<tr>
<td>Negative asymmetry</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC&lt;0%</td>
<td>-0.58*</td>
<td>-0.48*</td>
<td>-0.70*</td>
<td>-0.45*</td>
</tr>
<tr>
<td>Weak asymmetry</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CA =[-100%; 100%]</td>
<td>+0.24*</td>
<td>+0.16*</td>
<td>+0.33*</td>
<td>+0.35*</td>
</tr>
</tbody>
</table>

*Significant at 5%. Sample = 134 countries.

Figure 15 compares the scores for asymmetry of volatility with scores for kurtosis of volatility. A U-shaped relationship can be observed between the two measures, whose return point is around a zero value for the coefficient of asymmetry. Table 7 shows a very high positive correlation between these two indicators when the sample is limited to countries with a positive asymmetry for the distribution of exports, and a significant negative correlation for countries with a negative asymmetry for the distribution of exports. Moreover, an analysis of these correlations shows that a high kurtosis most frequently corresponds to a positive asymmetry for shocks, particularly for distributions around mixed trends and the HP(100) filter. Conversely, for degrees of asymmetry between 0 and 100% in absolute values, we see a very moderate positive relationship both in graphical terms and in terms of coefficients, suggesting that these two aspects of volatility are relatively independent of each other for these values.

A low skewness in absolute values therefore appears to correspond to small-magnitude, high-frequency variations, whilst a high coefficient of asymmetry seems to correspond to large-magnitude but lower frequency variations. It therefore becomes possible to see a bidimensional measure of instability in the coefficient of asymmetry of a distribution around its trend, reflecting both asymmetry and the occurrence of extreme deviations.
Figure 15. Correlation between asymmetry and kurtosis of volatility indicators, by reference value
Conclusion

The aim of this article is to review the research into macroeconomic volatility along with an analysis of the usual indicators of volatility. The literature on the subject, although very extensive, uses a diverse range of volatility indicators. The techniques used vary in terms of the choice of reference value and the way in which deviations from the reference value are calculated. Although, in general, the research is limited to examining the magnitude of volatility, it is possible to extend the analysis to the effects of the asymmetry of shocks and the occurrence of extreme deviations. In fact, both of these aspects are separate dimensions of volatility but largely ignored in the literature on the subject.

Moreover, we have limited our analysis to indicators designed to reflect the transitory variations - both certain and uncertain - in export revenues. Measuring the uncertainty resulting from the fluctuations of an economic variable requires the use of specific statistical tools, which have not been addressed in this paper. We have suggested four possible methods for calculating a reference value. The first two involve predicting the change in exports based on a mixed trend, including a deterministic and a random component. This method is reliant on selecting an appropriate functional form. Although observing the behaviour of the series for each country can be time-consuming when working on panel data, a thorough review of the literature on the order of integration of the variable concerned, combined with a spectrum analysis and the use of unit root tests can provide us with useful information on specifying an appropriate functional form. Moreover, in our view it is better to opt for a rolling estimated trend than a "global" mixed trend, since it enables the coefficients estimated to vary over time and reflects recent changes in the trend of the series more accurately.

A filtering approach has the advantage of not being based on an a priori formalisation of the change in the series. It also offers the advantage of being sensitive to breaks in the trend over time. Problems of leakage or compression can nevertheless introduce/exclude undesirable/relevant components in order to study the effects of short-term fluctuations. Applied to developing countries, the choice of a smoothing parameter between 6 and 10 initially appears preferable, since the trend in these countries seems to fluctuate more significantly.

We have also set out the various possibilities for calculating deviations and the various effects the latter are assumed to reflect. We have drawn a distinction between the magnitude of volatility - measured by an indicator of average dispersion around the reference value, asymmetry - measured by the coefficient of asymmetry for the distribution of deviations, and the occurrence of the extreme values of deviations - measured by the kurtosis of the distribution of deviations. Whilst the indicators for the magnitude of volatility obtained from the various reference values are very strongly correlated, this is not the case for measures of asymmetry and kurtosis. It thus appears that every moment in the distribution of deviations around the reference values expresses a particular aspect of volatility. The coefficient of asymmetry seems particularly interesting because of its
bidimensional nature. Not only does it express the degree of asymmetry of fluctuations around a trend over a given period, but it also reflects the frequency of extreme deviations.

Bibliography


Appendices
## Appendix A.1 Overview of indicators of volatility and their application in the literature.

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Authors</th>
<th>Phenomenon</th>
<th>Variables concerned (yt)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Growth rate/first difference</strong></td>
<td></td>
<td></td>
<td>GDP/inhabitant, inflation, terms of trade, actual exchange rate, black-market premium, international interest rate (LIBOR), development aid/inhabitant, public spending, ratio of wheat cultivation yields to national yield</td>
</tr>
<tr>
<td>Variance of growth rate over five years</td>
<td>Koren and Tenreyro (2006)</td>
<td>Variability</td>
<td>Annual growth rate of work productivity</td>
</tr>
<tr>
<td><strong>Filters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decline in GDP: decrease of more than 1% of the (annual) series log smoothed by the HP filter (lambda=1000)</td>
<td>Mauro and Becker (2006)</td>
<td>Variability</td>
<td>GDP</td>
</tr>
<tr>
<td>Standard deviation of the cyclical component, i.e. the standard deviation of the difference between series smoothed by the HP or BK filter and actual series.</td>
<td>Hnatkovska and Loayza (2005); Guillaumont and Chauvet (2007), Afonso and Furceri (2010)</td>
<td>Variability</td>
<td>Aid, export revenues, GDP.</td>
</tr>
<tr>
<td><strong>Variables concerned (yt)</strong></td>
<td></td>
<td></td>
<td>GDP, budget variables (transfers, subsidies, public spending, tax revenues, etc.)</td>
</tr>
</tbody>
</table>
Average over five years of the ratio of the absolute deviation between the observed value of export revenues \( (X) \) and the value filtered using a moving average process over five years, based on GDP \( (Y) \):

\[
INST = \frac{1}{T-4} \sum_{j=5}^{T} \left| \frac{1}{5} \sum_{k=j-2}^{j+2} X_k - \frac{1}{5} \sum_{k=j-2}^{j+2} Y_k \right|
\]

Dawe (1996) | Variability | Export revenues

\[\Delta y_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t \text{ dummy}_{394} + \beta_1 y_{t-1} + \gamma_1 \text{ dummy}_{394} + \delta D_j + \epsilon_t\]

The standard deviation of the residual \( \epsilon_t \), is seen as a measure of volatility reflecting uncertainty. This approach is adopted for the whole of the sample and for all countries, to produce an estimate of coefficients specific to each country.

Ramey and Ramey (1995) | Uncertainty | Growth rate of GDP

\[y_t = \alpha + \beta t + \epsilon_t\]

The standard deviation of the residual \( \epsilon_t \), obtained using a regression of \( y_t \) on a linear trend:

Pritchett (2000), Mobarak (2005) | Variability | Growth rate of GDP per inhabitant, growth rate of capital per worker

\[y_t = \alpha + \beta t + \gamma_1 y_{t-1} + \epsilon_t\]

Growth rate of GDP per inhabitant, growth rate of capital per worker | Variability/uncertainty | Development aid, exports, terms of trade, inflation rate, relative price of capital, actual exchange rate, growth rate of GDP.

\[y_t = \alpha + \beta t + \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \gamma_3 y_{t-3} + \epsilon_t\]

Standard deviation of the error in a regression of FDI over three lags and a temporal trend:

Lensink and Morrissey (2006) | Uncertainty | FDI/GDP, FDI

\[\Delta y_u = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t \text{ dummy}_{394} + \beta_1 \Delta y_{u-1} + \beta_2 y_{u-2} + \delta D_i + \epsilon_u\]

They estimate volatility measures for each country based on the following GARCH(1,1) model:


\[\sigma^2_u = \gamma_{1,0} + \gamma_{1,1} \epsilon_{t-1}^2 + \delta \sigma^2_{u-1}\]

The estimated value of \( \sigma_u^2 \) represents the uncertainty of \( y_u \).
Appendix A.2. Mixed trends and correlogram of residuals

Venezuela

South Korea

Burundi
Kenya

Exports (USD, 2000)

Global mixed trend

Rolling mixed trend

Autocorrelations

Lags

Bartlett’s formula for MA(q) 95% confidence bands
Appendix A.3. Correlogram of export revenue cycles smoothed by the HP filter

Argentina

Belize

Ivory Coast

Kenya

South Korea

Venezuela
Appendix A.4 Comparative evolution of deviations (in % of trend values)